

Exercise 62

If $h(x) = \sqrt{4 + 3f(x)}$, where $f(1) = 7$ and $f'(1) = 4$, find $h'(1)$.

Solution

Take the derivative of $h(x)$.

$$\begin{aligned}h'(x) &= \frac{d}{dx} \sqrt{4 + 3f(x)} \\&= \frac{1}{2} [4 + 3f(x)]^{-1/2} \cdot \frac{d}{dx} [4 + 3f(x)] \\&= \frac{1}{2} [4 + 3f(x)]^{-1/2} \cdot 3f'(x) \\&= \frac{3}{2} \frac{f'(x)}{\sqrt{4 + 3f(x)}}\end{aligned}$$

Set $x = 1$.

$$\begin{aligned}h'(1) &= \frac{3}{2} \frac{f'(1)}{\sqrt{4 + 3f(1)}} \\&= \frac{3}{2} \frac{4}{\sqrt{4 + 3(7)}} \\&= \frac{6}{5}\end{aligned}$$